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NUMERICAL SOLUTION OF UNSTEADY HEAT TRANSFER OVER A POROUS FLAT PLATE

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Numerical solution of the energy equation for the temperature distribution in viscous flow past a porous flat plate is obtained. A uniform suction, that follows step function change, is applied normal to the plate. The results for different values of Prandtl number are found by finite difference technique, when the suction velocity doubles in the step change.

1. INTRODUCTION

A number of problems concerning flow and heat transfer over a flat plate with suction has been analysed by several authors[2,6,7]. Schillichting[5] obtained an exact solution to the problem of uniform flow past a flat plate subject to uniform suction. Bhargava and Agarwal[1] has discussed the problem of unsteady flow past a flat plate by finite difference method when suction follows a step function change. In the present paper, the heat transfer part of the above problem is considered. A finite difference scheme using Crank Nikolson method is employed to find the numerical values of the temperature function. The stability criterion has already been satisfied. Within a few steps, results in good coincidence with the exact solution, are obtained.

2. FORMULATION OF THE PROBLEM

Consider the two dimensional flow of a viscous incompressible fluid past a semi infinite porous flat plate coinciding with the plane $y = 0$. The fluid flows with a constant velocity U_∞ parallel to the plate. The x and y axes are taken respectively along and normal to the plate and let u and v be the velocity component in these directions. A constant suction velocity $v_1 < 0$ is applied normal to the plate for time $t < 0$. At $t = 0$, this suction velocity is changed to $v_2 < 0$ and is maintained for all times $t > 0$. Initially the plate and fluid have the same temperature T_w and the fluid at infinity is assumed to have a constant temperature T_∞ for all times $t \geqslant 0$.

The energy equation is

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} \qquad ...(1)$$

where k is the coefficient of thermal conductivity.

Since flow function is independent of x, the distance parallel to the wall, accordingly the temperature function T is also independent of x, so (1) can be written as

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2}. \qquad ...(2)$$

The boundary conditions are,

$$\begin{aligned} &T(0, t) = T_w \\ &T(\infty, t) = T_\infty \\ &v(0, t) = v_1, \; t \leqslant 0 \\ &\qquad\quad = v_2, \; t > 0 \end{aligned} \qquad ...(3)$$

and the initial condition is

$$T(y, 0) = T_\infty + (T_w - T_\infty)\, e^{-v_1 y/\nu}. \qquad ...(4)$$

Introducing the dimensionless variables,

$$\eta = \frac{|v_1|\, y}{\nu}, \; \tau = \frac{|v_1|^2\, t}{\nu}, \; \bar{u} = \frac{u}{U_\infty}, \; \bar{v} = \frac{v}{U_\infty}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \qquad ...(5)$$

equation (2) is transformed to,

$$\frac{\partial \theta}{\partial \tau} + \frac{v}{|v_1|}\frac{\partial \theta}{\partial \eta} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial \eta^2}. \qquad ...(6)$$

The corresponding boundary conditions are

$$\begin{aligned} &\theta(0, \tau) = 1 \\ &\theta(\infty, \tau) = 0 \\ &\frac{v}{|v_1|} = -1, \; \tau \leqslant 0 \\ &\qquad\quad = -\lambda, \; \tau > 0; \end{aligned} \qquad ...(7)$$

and the initial condition is

$$\theta(\eta, 0) = e^{-\eta}. \qquad \ldots(8)$$

The numerical value of λ has been taken as 2 in the analysis. However, the method works for any value of λ.

3. Method of Solution

Defining a new dimensionless variable,

$$\eta' = \frac{\eta}{1+\eta} \qquad \ldots(9)$$

the governing equation (6) takes the form

$$Pr \frac{\partial \theta}{\partial \tau} = -2(1-\eta')^2 \{1-\eta'-Pr\} \frac{\partial \theta}{\partial \eta'} + (1-\eta')^4 \frac{\partial^2 \theta}{\partial \eta'^2} \qquad \ldots(10)$$

with

$$\theta(0, \tau) = 1,$$

$$\theta(1, \tau) = 0 \qquad \ldots(11)$$

and

$$\theta\left(\frac{\eta'}{1-\eta'}, 0\right) = e^{-\eta'/(1-\eta')}.$$

For convenience, dropping dashes, eqn. (10) becomes

$$Pr \frac{\partial \theta}{\partial \tau} = -2(1-\eta)^2 \{1-\eta-Pr\} \frac{\partial \theta}{\partial \eta} + (1-\eta)^4 \frac{\partial^2 \theta}{\partial \eta^2} \qquad \ldots(12)$$

with

$$\theta(0, \tau) = 1,$$

$$\theta(1, \tau) = 0$$

and

$$\theta\left(\frac{\eta}{1-\eta}, 0\right) = e^{-\eta/(1-\eta)}. \qquad \ldots(13)$$

4. Finite Difference Scheme

An implicit finite difference method has been used to solve eqn. (12) subject to (13). The flow region is considered as a semi infinite strip bounded by $\eta = 0$ and $\eta = 1$. To obtain the difference equations, the region is divided into a grid or mesh of lines parallel to η and τ axes (Fig. 1). The spacing between the lines parallel to η-axis is chosen arbitrarily while spacing between the lines parallel to τ axis is taken so as to satisfy the stability condition as per Ralston[4]. Solution of the difference equation is obtained at the intersection of these mesh lines, called nodes. The values of the dependent variables θ at the nodel points along the plane $\eta = 0$ are given by $\theta\,(0, \tau)$ and thus are known.

The following difference scheme is followed :

$$\frac{\partial \theta}{\partial \tau} = \frac{\theta_{i,\,j+1} - \theta_{i,\,j}}{\Delta \tau}$$

$$\frac{\partial \theta}{\partial \eta} = \frac{\theta_{i+1,j} - \theta_{i,j}}{\Delta \eta} \qquad \ldots(14)$$

$$\frac{\partial^2 \theta}{\partial \eta^2} = \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{(\Delta \eta)^2} .$$

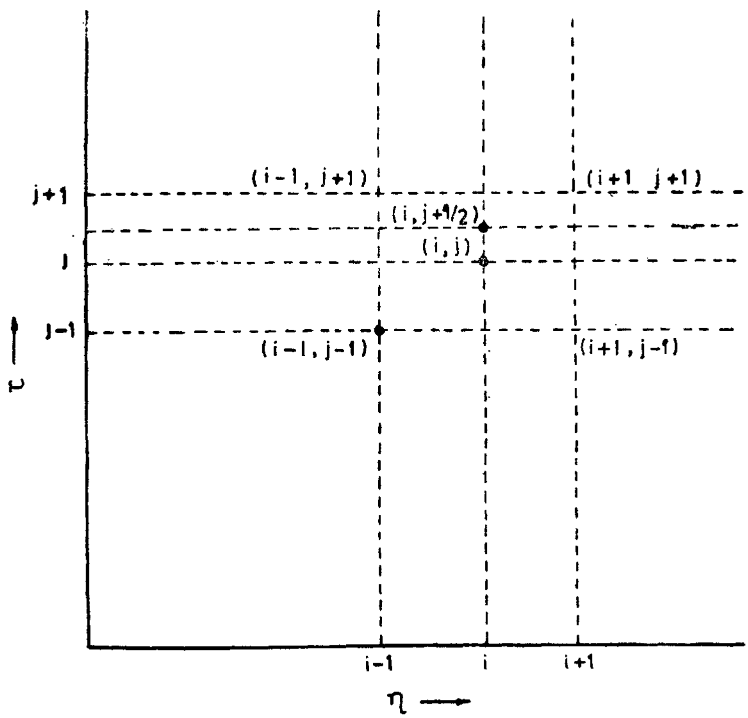

Fig. 1.

Using Crank-Nikolson method, eqn. (12) becomes

$$Pr\left[\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta\tau}\right] = -(1-\eta_{i,j})^2\{1-\eta_{i,j}-Pr\}\left[\frac{\theta_{i+1,j+1}-\theta_{i,j+1}}{\Delta\eta}\right.$$

$$\left.+\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta\eta}\right]+\frac{1}{2}(1-\eta_{i,j})^4$$

$$\times\left[\frac{\theta_{i+1,j+1}-2\theta_{i,j+1}-\theta_{i-1,j+1}}{(\Delta\eta)^2}\right.$$

$$\left.+\frac{\theta_{i+1,j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta\eta)^2}\right]. \qquad \ldots(15)$$

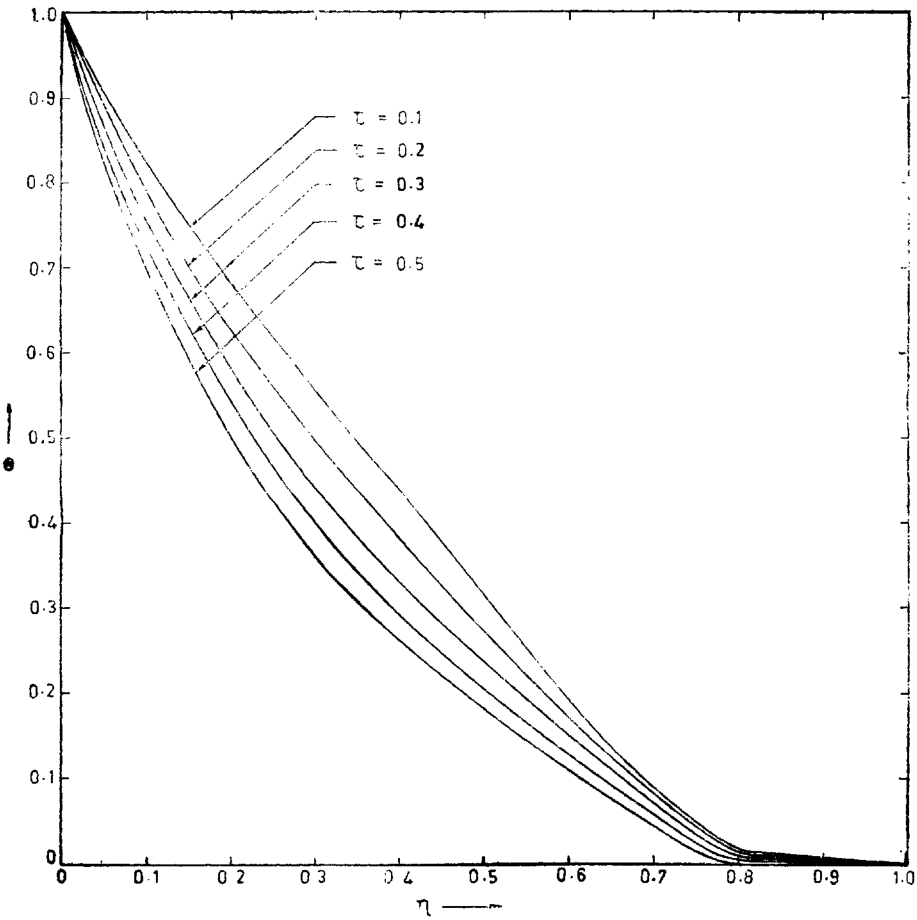

FIG. 2. Variation of θ with η for different τ and $Pr = 2.0$.

The conditions (13) now, are

$$\theta_{i,0} = \exp\left(-(i-1)\,\Delta\eta/(1-(i-1)\,\Delta\eta)\right),\ i = 2, \ldots, 10$$

$$\theta_{1,j} = 1.0 \qquad \ldots(16)$$

$$\theta_{11,j} = 0.0$$

From eqn. (15), a set of simultaneous equations has been obtained whose solution is tried by Gaussian elimination method.

The Nusselt number on the plate is given by

$$Nu = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

and has been shown in Fig. 4.

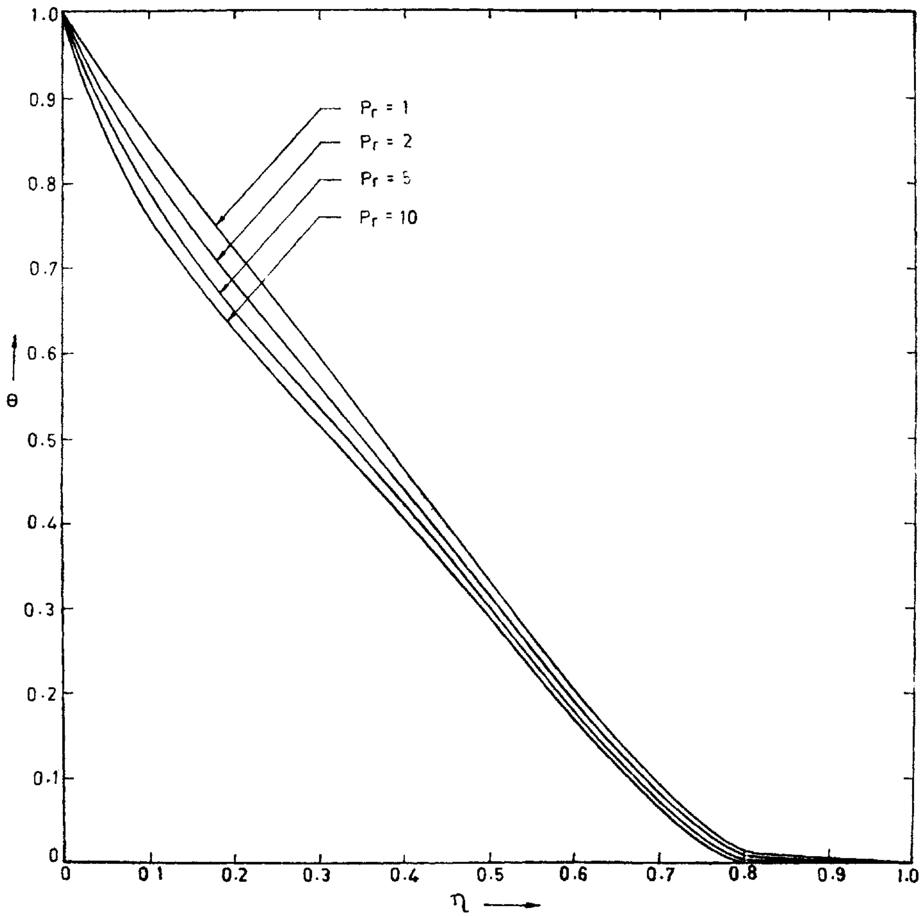

FIG. 3. Variation of θ with η for different Pr and $\tau = 0.1$.

5. Discussion of Results

Temperature distribution has been shown in Figs. 1 and 2 for different values of time τ and Prandtl number Pr. In all these calculations the values of $\Delta\eta$ has been chosen to be 0.1 and $\Delta\tau$ is taken as .005 satisfying stability conditions. Fig. 2 reveals that for a fixed $Pr = 2.0$, the temperature decreases throughout with increase in τ. This decrease in temperature is more apparent near the first quarter of the gap length η. Figure 3 gives the variation of temperature distribution for different Pr and for a fixed $\tau = 0.1$. It is interesting to note that the temperature decreases more rapidly near the plate with increase in Pr.

This difference scheme is more consistent as compared to other one. The truncation error for the approximation is $O(\Delta\tau)^2 + O(\Delta\eta)^2$ which tends to zero as $\Delta\eta$ and $\Delta\tau$ both tending to zero. We have also calculated the values of temperature for different τ and Pr from the exact solution of the differential equation obtained by Purohit and Goyal[3] using Laplace transform. The two results viz. (i) from finite

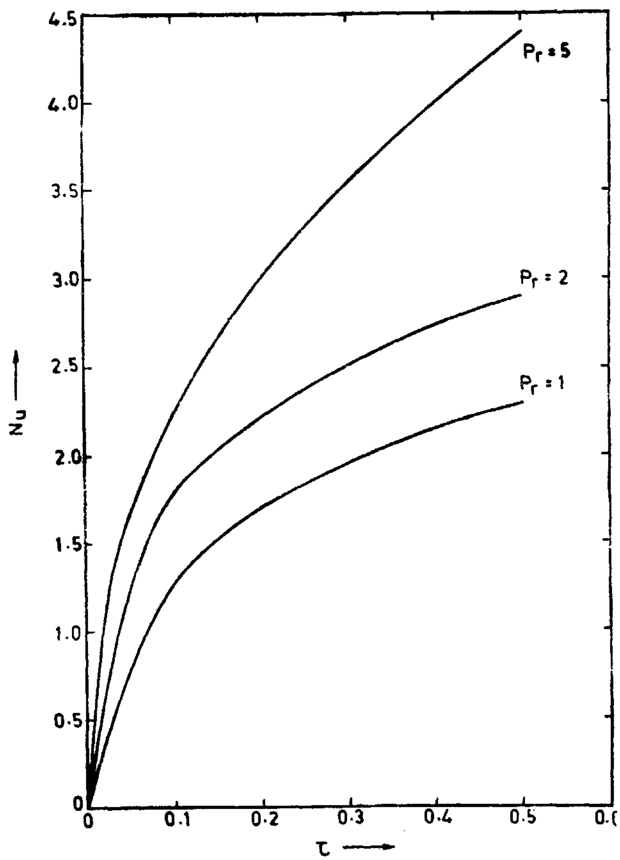

Fig. 4. Variation of nusselt number with τ for different Pr.

TABLE I

η \ τ	$P_r = 2.0$									
	0.1		0.2		0.3		0.4		0.5	
	F. D.	L. T.	F. D.	L. T.	F. D.	L. T.	F. D.	L. T.	F.D.	L.T.
0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.1	.82468	.82775	.78624	.78139	.75773	.74878	.73491	.73500	.71593	.71432
0.2	.68652	.68925	.62545	.61994	.57863	.56317	.54061	.54922	.50872	.50062
0.3	.56347	.57492	.49702	.49310	.44396	.43952	.39998	.38908	.36263	.35993
0.4	.44214	.43921	.38389	.38591	.33551	.33092	.29449	.29054	.25912	.25192
0.5	.31805	.31185	.27572	.26984	.23946	.22871	.20803	.19356	.18051	.17335
0.6	.19530	.18992	.17109	.17004	.14986	.14578	.13108	.13524	.11435	.10985
0.7	.08734	.08534	.07862	.07683	.07069	.06732	.06348	.06212	.05689	.05361
0.8	.01748	.01731	.01666	.01674	.01585	.01501	.01505	.01500	.01427	.01399
0.9	.00013	.00012	.00011	.00011	.00011	.00011	.00010	.00008	.00009	.00003
1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

TABLE II

η \ P_r	$\tau = .1$							
	1.0		2.0		5.0		10.0	
	F. D.	L. T	F. D.	L. T.	F. D.	L T	F. D	L.T.
0.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
0.1	.86074	.85317	.82468	.82775	.79256	.78139	.75926	.74393
0.2	.72810	.71184	.68652	.68925	.65325	.64450	.63148	.62991
0.3	.59811	.59211	.56347	.57492	.54197	.54100	.51848	.50019
0.4	.46728	.46251	.44214	.43921	.42790	.40316	.41345	.39122
0.5	.33461	.33721	.31805	.31185	.30868	.30186	.29565	.29129
0.6	.20459	.19949	.19530	.18992	·18297	.17003	.17824	.16311
0.7	.09099	.09313	.08734	.08534	.07524	.06128	.06749	.05914
0.8	.01808	.01731	.01748	.01731	.01714	.01634	.01693	.01521
0.9	.00014	.00018	.00013	.00011	.00012	.00010	.00011	.00007
1.0	0.0	0.0	0.0	0·0	0.0	0.0	0.0	0.0

difference method (ii) from exact solution for different τ and Pr match quite satisfactorily (see Tables I and II).

The Nusselt number on the plate has been calculated for different τ. From Fig. 4, it is evident that more heat is transformed from the plate in the beginning but it appears to reach a steady value with passing up of time. Also the phenomenon is more pronounced for a large Prandtl number.

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