Indian J. pure appl. Math., 19 (11) : 1043-1053, November 1988

ORDER LEVEL INVENTORY SYSTEM WITH POWER DEMAND PATTERN FOR ITEMS WITH VARIABLE RATE OF DETERIORATION

T. K. DATTA AND A. K. PAL

Department of Mathematics, Jadavpur University, Calcutta 700032

(*Received* 24 *July* 1987; *after revision* 27 *November* 1987)

The present paper deals with a power demand pattern inventory model with variable rate of deterioration. Both deterministic and probabilistic demands have been considered. Ultimately, some particular cases regarding the demand pattern have also been discussed.

1. INTRODUCTION

The effect of deterioration is very important in many inventory systems. Deterioration is defined as decay or damage such that the item can not be used for its original purpose. Food items, drugs, pharmaceuticals and radioactive substances are examples of items in which sufficient deterioration can take place during the normal storage period of the units and consequently this loss must be taken into account when analyzing the system. Efforts in analyzing mathematical models of inventory in which a constant or variable proportion of the on-hand inventory deteriorates with time have been undertaken by Ghare and Schrader[1], Goel and Aggarwal[2], Covert and Philip[3], Shah[4], Misra[5] etc. to name only a few. Covert and Philip in their paper have developed an economic order quantity model for items with variable rate of deterioration by assuming Weibull density function for the time of deterioration and a constant demand rate.

In the present paper attempts have been made to investigate an EOQ model assuming the existance of a suitable power demand pattern and a special form of Weibull density function. Such a special form is chosen in order to make the problem mathematically tractable. Deterministic as well as probabilistic cases of demands are considered allowing shortages. Ultimately, some particular cases of the power demand pattern have been discussed.

2. THE MATHEMATICAL MODEL

The model is developed under the following assumptions :

(i) Replenishment size is constant and the replenishment rate is infinite;

(ii) lead time is zero;

(iii) T is the fixed length of each production cycle;

(iv) C_1 is the inventory holding cost per unit per unit time;

(v) C_2 is the shortage cost per unit per unit time;

(vi) C_3 is the cost of each deteriorated unit;

(vii) shortages are allowed and fully backlogged;

(viii) a variable fraction $\theta(t)$ of the on-hand inventory deteriorates per unit time.

In the present model, the function $\theta(t)$ is assumed in the form

$$\theta(t) = \theta_0 t;\ 0 < \theta_0 \leqslant 1,\ t > 0$$

which is a special form of two parameter Weibull function considered by Covert and Philip[3];

(ix) the demand upto time t is assumed to be $d\left(\frac{t}{T}\right)^{1/n}$, vide, Naddor[6] where d is the demand size during the fixed cycle time T and n $(0 < n < \infty)$ is the pattern index. $(dt^{(1-n)/n})/nT^{1/n}$ is the demand rate at time t. Such pattern in the demand rate is called power demand pattern.

3. DETERMINISTIC DEMAND

Let Q be the total amount of inventory produced or purchased at the beginning of each period and after fulfilling backorders let us assume we get an amount S $(S > 0)$ as initial inventory. Let d be the demand during period T. Inventory level gradually diminishes during time period $(0, t_1)$, $t_1 < T$ due to the reasons of market demand and deterioration of the items and ultimately falls to zero at time $t = t_1$. Shortages occur during time period (t_1, T) which are fully backlogged. Let $I(t)$ be the on-hand inventory at any time t. The differential equations which the on-hand inventory $I(t)$ must satisfy in two different parts of the cycle time T are the following:

$$\frac{dI(t)}{dt} + \theta(t)\, I(t) = -\frac{dt^{(1-n)/n}}{nT^{1/n}},\ 0 \leqslant t \leqslant t_1 \qquad \text{...(3.1)}$$

and

$$\frac{dI(t)}{dt} = -\frac{dt^{(1-n)/n}}{nT^{1/n}},\ t_1 \leqslant t \leqslant T. \qquad \text{...(3.2)}$$

Solutions of the above differential equations are

$$I(t) = S\exp\left(-\frac{\theta_0}{2}t^2\right) - \frac{d\exp\left(-\frac{\theta_0}{2}t^2\right)}{nT^{1/n}}\int_0^t t^{(1-n)/n} \times \exp\left(\frac{\theta_0}{2}t^2\right)dt,\ 0 \leqslant t \leqslant t_1 \qquad \text{...(3.3)}$$

and

$$I(t) = \frac{d}{T^{1/n}}\left(t_1^{1/n} - t^{1/n}\right),\ t_1 \leqslant t \leqslant T. \qquad \ldots(3.4)$$

Since $I(t_1) = 0$, we find neglecting higher order terms of θ_0 ($\ll 1$) the following :

$$S = \frac{dt_1^{1/n}}{T^{1/n}} + \frac{\theta_0 d}{2(2n+1)T^{1/n}}\, t_1^{(1+2n)/n}. \qquad \ldots(3.5)$$

Hence the total amount of deteriorated units

$$= S - \int_0^{t_1} \frac{dt^{(1-n)/n}}{nT^{1/n}}\, dt = S - \frac{dt_1^{1/n}}{T^{1/n}} = \frac{\theta_0\, dt_1^{(1+2n)/n}}{2(2n+1)T^{1/n}}. \qquad \ldots(3.6)$$

Average total cost per unit time is given by

$$C(S, t_1) = \frac{C_3\, \theta_0\, dt_1^{(1+2n)/n}}{2T^{(1+n)/n}(2n+1)} + \frac{C_1}{T}\int_0^{t_1} I(t)\, dt - \frac{C_2}{T}\int_{t_1}^{T} I(t)\, dt.$$

Now substituting the values for $I(t)$ given by eqns (3.3), (3.4), eliminating S using eqn. (3.5) and then on integration we find

$$\begin{aligned} C(t_1) = {} & \frac{C_3\, \theta_0 d}{2(2n+1)T^{(1+n)/n}}\, t_1^{(1+2n)/n} + \frac{C_1 d}{T^{(1+n)/n}}\, t_1^{(1+n)/n} \\ & + \frac{C_1\, \theta_0\, d}{2(2n+1)\, T^{(1+n)/n}}\, t_1^{(1+3n)/n} - \frac{C_1\, \theta_0\, d}{6T^{(1+n)/n}}\, t_1^{(1+3n)/n} \\ & - \frac{C_1\, nd}{(n+1)T^{(1+n)/n}}\, t_1^{(1+n)/n} + \frac{C_1\, n^2\, \theta_0\, d}{(2n+1)(3n+1)T^{(1+n)/n}}\, t_1^{(1+3n)/n} \\ & + \frac{C_2\, nd}{n+1} - \frac{C_2\, d}{T^{1/n}}\, t_1^{1/n} + \frac{C_2\, d}{(n+1)T^{(1+n)/n}}\, t_1^{(1+n)/n}. \end{aligned}$$

[neglecting terms containing higher powers of θ_0]. ...(3.7)

For minimum, the necessary condition is

$$\frac{dC(t_1)}{dt_1} = 0.$$

This gives

$$\frac{t_1^{(1-n)/n}}{T^{(1+n)/n}}\left[\frac{C_3\, \theta_0\, d}{2n}\, t_1^2 + \frac{C_1(n+1)d}{n}\, t_1 + \frac{C_1\, \theta_0\,(3n+1)\, d}{2n(2n+1)}\, t_1^3 \right.$$

(*equation continued on p.* 1046)

$$- C_1\, dt_1 + \frac{C_1\, n\, \theta_0\, d}{2n + 1} t_1^3 - \frac{C_1\, \theta_0\, (3n + 1)\, d}{6n} t_1^3 - \frac{C_2\, dT}{n}$$

$$+ \frac{C_2\, d}{n} t_1 \Big] = 0.$$

But as $t_1 > 0$ we find from the above equation the following cubic in t_1.

$$Lt_1^3 + M\, t_1^2 + Nt_1 + P = 0 \qquad \text{...(3.8)}$$

where

$$\left.\begin{aligned} L &= \frac{C_1\, \theta_0\, d}{3n}, \\ M &= \frac{C_3\, \theta_0\, d}{2n}, \\ N &= \frac{d}{n}(C_1 + C_2), \\ P &= -\frac{C_2\, dT}{n} \end{aligned}\right\} . \qquad \text{...(3.9)}$$

Since (3.8) is a cubic equation in t_1 with last term P negative, it has at least one positive root. Again since $0 < \theta_0 \lll 1$, we find $LN - M^2 > 0$ and so other two roots of (3.8) are imaginary. Let t_{10} be the positive root of (3.8).

$\therefore$ Optimum value of t_1 is $t_1^* = t_{10}$. Substituting it in (3.5), the optimum S is

$$S^* = \frac{d t_1^{*1/n}}{T^{1/n}} + \frac{\theta_0 d}{2\,(2n+1)\, T^{1/n}}\, t_1^{*(1+2n)/n} . \qquad \text{...(3.10)}$$

The relation connecting S and Q is the following

$$Q = S + d - \frac{d t_1^{1/n}}{T^{1/n}}. \qquad \text{...(3.11)}$$

Hence the optimum value for Q is

$$Q^* = d + \frac{\theta_0\, d}{2\,(2n + 1) T^{1/n}}\, t_1^{*(1+2n)/n}. \qquad \text{...(3.12)}$$

Minimum value of C is $C\left(t_1^*\right)$.

If there be no deterioration, then $\theta_0 = 0$.

$\therefore$ From (3.8) [using (3.9)]

$$t_1^* = \frac{C_2 T}{C_1 + C_2}$$

and corresponding expressions for Q^* and S^* can be obtained by substituting $\theta_0 = 0$ in the expressions (3.12) and (3.10) respectively.

4. Probabilistic Demand

In this case, it is assumed that the demand during the period $(0, T)$ is a random variable x with probability density function $f(x)$ $(0 < x < \infty)$ and the demand follows power demand pattern with the demand rate $(xt^{(1-n)/n})/nT^{1/n}$. Solution of the problem has been investigated under the following two cases.

Case 1 : *When no shortages occur*

Let us assume the inventory level of the system at any time t $(0 \leqslant t \leqslant T)$ to be $I_{1x}(t)$. Hence the differential equation which would govern the system would be

$$\frac{dI_{1x}(t)}{dt} + \theta_0\, t I_{1x}(t) = -\,\frac{xt^{(1-n)/n}}{nT^{1/n}},\ 0 \leqslant t \leqslant T. \qquad \ldots(4.1)$$

Solution of the equation (4.1) is the following :

$$I_{1x}(t) = S\exp\left(-\frac{\theta_0}{2}t^2\right) - \frac{x\exp\left(-\frac{\theta_0}{2}t^2\right)}{nT^{1/n}}\int_0^t t^{(1-n)/n}\exp\left(\frac{\theta_0}{2}t^2\right)dt,$$

$$0 \leqslant t \leqslant T \qquad \ldots(4.2)$$

where $S\ (>0)$ is the expected stock on hand at the beginning after meeting backorders. Since there is no shortage, we have

$$I_{1x}(T) \geqslant 0$$

or,

$$S - \frac{x}{n\,T^{1/n}}\int_0^T t^{(1-n)/n}\exp\left(\frac{\theta_0}{2}t^2\right)dt \geqslant 0$$

or,

$$x \leqslant S_1,$$

where

$$S_1 = \frac{SnT^{n/1}}{\int_0^T t^{(1-n)/n}\exp\left(\frac{\theta_0}{2}t^2\right)dt}.$$

The expression for S_1 can be simplified further by neglecting higher powers of θ_0. We ultimately obtain

$$S_1 = S\left[1 - \frac{\theta_0 T^2}{2(2n+1)}\right]. \qquad \text{...(4.3)}$$

The average number of items $H_1(x)$ carried in inventory per unit time is the following

$$H_1(x) = \frac{1}{T}\int_0^T I_{1x}(t)\,dt,\ x \leqslant S_1$$

$$= S - \frac{S\theta_0 T^2}{6} - \frac{nx}{n+1} + \frac{x\,\theta_0\, n^2 T^2}{(2n+1)(3n+1)},\ x \leqslant S_1. \qquad \text{...(4.4)}$$

Average number of items that deteriorates per unit time is

$$D_1(x) = \frac{1}{T}[S - x - I_{1x}(T)]$$

$$= \tfrac{1}{2} S\theta_0 T - \frac{n\,\theta_0\,T}{2n+1}\,x,\ x \leqslant S_1. \qquad \text{...(4.5)}$$

Average shortage per unit time is

$$G_1(x) = 0. \qquad \text{...(4.6)}$$

Case 2 : *When shortages occur*

If the system carries inventory during the period $(0, t_1)$ and then shortages occur for the remaining period (t_1, T) of the cycle then the inventory level $I_{2x}(t)$ at any instant t would satisfy similar differential equations as in the deterministic case and their solutions would be

$$I_{2x}(t) = S\exp\left(-\frac{\theta_0}{2}t^2\right) - \frac{x\exp\left(-\frac{\theta_0}{2}t^2\right)}{nT^{1/n}}\int_0^t t^{(1-n)/n}\exp\left(\frac{\theta_0}{2}t^2\right)dt,$$

$$0 \leqslant t \leqslant t_1 \qquad \text{...(4.7)}$$

and

$$I_{2x}(t) = \frac{x}{T^{1/n}}\left(t_1^{1/n} - t^{1/n}\right),\ t_1 \leqslant t \leqslant T. \qquad \text{...(4.8)}$$

Since shortages occur, we must have

$$I_{2x}(T) < 0$$

or

$x > S_1$, where S_1 is given in (4.3).

Again, $I_{2x}(t_1) = 0$. This gives

$$S - \frac{x}{nT^{1/n}} \int_0^{t_1} t^{(1-n)/n} \exp\left(\frac{\theta_0}{2} t^2\right) dt = 0.$$

Expanding the integrand in ascending powers of θ_0 and then integrating and neglecting all higher order terms in θ_0 we get

$$t_1^{1/n}\left[1 + \frac{\theta_0}{2(2n+1)} t_1^2\right] = \frac{S}{x} T^{1/n}. \qquad \text{...(4.9)}$$

Taking nth power of both sides and then simplifying we find

$$t_1\left[1 + \frac{n\theta_0}{2(2n+1)} t_1^2\right] = \left(\frac{S}{x}\right)^n T$$

or

$$t_1^3 + 2Ht_1 + G = 0 \qquad \text{...(4.10)}$$

where

$$H = \frac{2(2n+1)}{3\theta_0 n}, \; G = -\frac{2(2n+1)}{n\theta_0}\left(\frac{S}{x}\right)^n T.$$

Solving the above cubic equation (4.10) by Cardon's method we find the positive real root as follows :

$$t_1 = u - \frac{2(2n+1)}{3n\,\theta_0} \frac{1}{u}, \qquad \text{...(4.11)}$$

where

$$u^3 = \tfrac{1}{2}[-G + \sqrt{(G^2 + 4H^3)}]$$

$$= \tfrac{1}{2}\left[\frac{2\ (2n+1)}{n\theta_0} .T\left(\frac{S}{x}\right)^n + \left\{\frac{4(2n+1)^2 T^2}{n^2\,\theta_0^2}\left(\frac{S}{x}\right)^{2n} + \frac{32}{27}\frac{(2n+1)^3}{n^3\,\theta_0^2}\right\}^{1/2}\right].$$

Expressing u and $\frac{1}{u}$ in a series in ascending powers of θ_0 and then substituting them in the equation(4.11) we find

$$t_1 = \left(\frac{S}{x}\right)^n T - \frac{21}{16}\left(\frac{n\theta_0}{2n+1}\right) T^3 \left(\frac{S}{x}\right)^{3n}, \qquad \text{...(4.12)}$$

neglecting all higher order terms in θ_0. From the above equation the expressions for $t_1^{1/n}$, $t_1^{(1+n)/n}$, $t_1^{(1+3n)/n}$ etc. which are needed to calculate the total average expected cost can be obtained as follows :

$$t_1^{1/n} = \frac{S}{x} T^{1/n} - \frac{21}{16}\frac{\theta_0}{(2n+1)} T^{(1+2n)/n}\left(\frac{S}{x}\right)^{3n} \quad \text{...(4.13)}$$

$$t_1^{(1+n)/n} = \left(\frac{S}{x}\right)^{n+1} T^{(1+n)/n} - \frac{21}{16}\frac{(n+1)\,\theta_0}{(2n+1)} T^{(1+3n)/n}\left(\frac{S}{x}\right)^{3n+1} \quad \text{...(4.14)}$$

$$t_1^{(1+3n)/n} = \left(\frac{S}{x}\right)^{3n+1} T^{(1+3n)/n} - \frac{21}{16}\left(\frac{3n+1}{2n+1}\right)\theta_0\, T^{(1+5n)/n}\left(\frac{S}{x}\right)^{5n+1}. \quad \text{...(4.15)}$$

The following expression for S in terms of t_1 can be obtained from eqn. (4.9)

$$S = \left[\frac{t_1^{1/n}}{T^{1/n}} + \frac{\theta_0\, t_1^{(1+2n)/n}}{2\,(2n+1)T^{1/n}}\right] . \, x. \quad \text{...(4.16)}$$

The average number of items $H_2(x)$ carried in inventory per unit time is the following:

$$H_2(x) = \frac{1}{T}\int_0^{t_1} I_{2x}(t)\, dt,\ x > S_1$$

$$= \frac{x}{T^{(1+n)/n}}\left[\frac{1}{n+1}\, t_1^{(1+n)/n} + \frac{\theta_0}{3\,(3n+1)}\, t_1^{(1+3n)/n}\right],\ x > S_1. \quad \text{...(4.17)}$$

Average amount of inventory that deteriorates per unit time is

$$D_2(x) = \frac{1}{T}\left[S - \int_0^{t_1} \frac{x t^{(1-n)/n}}{nT^{1/n}}\, dt\right],\ x > S_1$$

$$= \frac{S}{T} - \frac{x t_1^{1/n}}{T^{(1+n)/n}}. \quad \text{...(4.18)}$$

Average shortages per unit time is

$$G_2(x) = = -\frac{1}{T}\int_{t_1}^{T} I_{2x}(t)\, dt,\ x > S_1$$

$$= \frac{x}{T^{(1+n)/n}}\left[\frac{n}{n+1} T^{(1+n)/n} - T t_1^{1/n} + \frac{1}{n+1}\, t_1^{(1+n)/n}\right].$$

$$x > S_1 \quad \text{...(4.19)}$$

$\therefore$ Expected total cost of the system per unit time becomes [using (4.3) for S_1]

$$K(t_1, S) = C_3 \int_0^{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right]} D_1(x) f(x)\, dx + C_3$$

$$\times \int_{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right]}^{\infty} D_2(x) f(x)\, dx + C_1$$

$$\times \int_0^{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right]} H_1(x) f(x)\, dx$$

$$+ C_1 \int_{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right)}^{\infty} H_2(x) f(x)\, dx$$

$$+ C_2 \int_0^{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right]} G_1(x) f(x)\, dx$$

$$+ C_2 \int_{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right]}^{\infty} G_2(x) f(x)\, dx.$$

Now substituting the values of $D_1(x)$, $D_2(x)$, $H_1(x)$, $H_2(x)$, $G_1(x)$, $G_2(x)$ from (4.5), (4.18), (4.4), (4.17), (4.6), (4.19) respectively and finally eliminating t_1 using (4.12), (4.13) (4.14) (4.15) we get the following :

$$K(S) = C_3 \int_0^{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right]} \left[\tfrac{1}{2} S\theta_0 T - \frac{n\theta_0 T}{2n+1} x\right] f(x)\, dx$$

$$+ C_3 \int_{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right.}^{\infty} \left[\frac{21}{16}\, \frac{\theta_0}{2n+1}\, T \frac{S^{3n}}{x^{3n-1}}\right]$$

$$\times f(x)\, dx$$

$$+ C_1 \int_0^{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right]} \left[S - \frac{S\theta_0 T^2}{6} - \frac{nx}{n+1}\right.$$

$$\left. + \frac{\theta_0 n^2 T^2}{(2n+1)(3n+1)} x\right] f(x)\, dx$$

(*equation continued on p.* 1052)

$$+ C_1 \int_{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right]}^{\infty} \left[\frac{1}{n+1}\frac{S^{n+1}}{x^n} - \frac{(157n+47)\,\theta_0}{48\,(2n+1)\,(3n+1)} T^2 \frac{S^{3n+1}}{x^{3n}}\right] f(x)\,dx$$

$$+ C_2 \int_{S\left[1-\frac{\theta_0 T^2}{2(2n+1)}\right]}^{\infty} \left[\frac{nx}{n+1} - S + \frac{21\theta_0 T^2}{16\,(2n+1)} \frac{S^{3n}}{x^{3n-1}} + \frac{1}{n+1}\frac{S^{n+1}}{x^n} - \frac{21\theta_0 T^2}{16\,(2n+1)}\frac{S^{3n+1}}{x^{3n}}\right] f(x)\,dx. \qquad \ldots(4.20)$$

If the probability density function $f(x)$ and pattern index n are prescribed, then right-hand side of eqn. (4.20) can be evaluated. The necessory condition for the minimum expected cost $K(S)$ is the relation

$$\frac{dK(S)}{dS} = 0.$$

Equating $\frac{dK(S)}{dS}$ to zero, the optimum value of $S = S^*$ (> 0) can be derived. For this value of $S = S^*$, the sufficient condition for minimum $\left.\frac{d^2 K(S)}{ds^2}\right|_{S=S^*} > 0$ would also be satisfied.

5. Discussion

In the present problem, a power demand pattern has been assumed with demand rate $(dt^{(1-n)/n})/nT^{1/n}$, where T, d, t are prescribed cycle time, entire demand during $(0, T)$ period, t is time $(0 \leqslant t \leqslant T)$ respectively and n is pattern index. Substituting different values to the pattern index n in the equations for the total cost and total expected cost per unit time given by the equations (3.7) and (4.20) respectively, we can determine the corresponding cost equations. Then differentiating and proceeding in the usual manner the optimum values S^*, t_1^* Q^* etc., can be evaluated. Substituting $n = 0$ and $n = \infty$ in the power demand pattern formula it can be seen that these two correspond to the two extreme cases, i. e., when the entire demand occurs at the end of the period and when the demand is instantaneous in nature. For $n = 1$ the demand pattern is uniform and if there is no deterioration then it corresponds to the case discussed in Wilson's model. In this case the total cost per unit time (deterministic case) given by (3.7) reduces to [by substituting $n = 1$, $\theta_0 = 0$]

$$C(t_1) = \frac{C_3 d}{T} + \frac{C_2 d}{2} + (C_1 + C_2)\frac{d}{2T^2} t_1^2 - \frac{C_2 d}{T} t_1.$$

For optimum C, $\frac{dC}{dt_1} = 0$.

Equating the derivative to zero and simplifying we find the optimum t_1 as

$$t_1 = t_1^* = \frac{C_2 T}{C_1 + C_2}.$$

Since $\left[\frac{d^2 C}{dt_1^2}\right]_{t_1 = t_1^*} > 0$, C would be minimized for $t_1 = t_1^*$.

For $n = 1$, $\theta_0 = 0$ we find from equation (3.10)

$$S^* = \frac{dt_1^*}{T} = \frac{d}{T}\frac{C_2 T}{C_1 + C_2} = \frac{C_2 d}{C_1 + C_2}.$$

Similarly, the expected total cost per unit time (probabilistic case) given by (4.20) reduces to

$$K(S) = C_1 \int_0^S \left[S - \frac{x}{2}\right] f(x)\,dx + C_1 \int_S^\infty \frac{S^2}{2x} f(x)\,dx$$

$$+ C_2 \int_S^\infty \left[\frac{x}{2} - S + \frac{S^2}{2x}\right] f(x)\,dx.$$

Solving $\frac{dK(S)}{dS} = 0$ we can determine the value of S^*.

When $n > 1$, a larger portion of the demand occurs towards the beginning of the period and when $0 < n < 1$, a larger portion of the demand occurs at the end of the period.

REFERENCES

1. P. M. Ghare and G. F. Schrader, *J. Ind. Engng.* **14** (1983), 238-43.
2. V. P. Goel and S. P. Aggarwal, *Proceedings All India Seminar on Operational Research and Decision Making*, March 1981.
3. R. P. Covert and G. C. Philip, *AIIE Trans*, 5 (1973), 323-26.
4. Y. K. Shah, *AIIE Trans.* 9 (1977) 108-12.
5. R. B. Misra, *Int. J. Prod. Res.* 13 (1975), 495-505.
6. E. Naddor, *Inventory Systems*. John Wiley and Sons, New York, 1966.